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Magnetic gradiometry: a new method for magnetic gradient measurements

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Abstract

Instead of using a pair of different magnetic field sensors and differentiating their outputs in order to derive a magnetic gradient value, a single sensitive element can be used for measuring directly a magnetic gradient. This is done by the use of a stiff metallic string clamped at both ends and pumped with an AC current the frequency of which is tuned to the second eigenmode of the string. The string is excited at that eigenmode in the presence of a quasi-static magnetic gradient. Then, the corresponding mechanical displacements of the string can be measured by the use of an inductive technique with an instrumental noise envelope of less than $\pm 10^{-12}$ m, per 1 s measurement interval.
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1. Introduction

Magnetic gradiometry is a powerful tool for many applications where tensor characteristics of magnetic fields produced by magnetic anomaly sources are of importance. Over the past decade a number of different gradiometer schemes have been employed, aimed at excluding a uniform magnetic background from the process of measurements. Almost all of them utilise two different magnetometers (or different pairs of magnetometers for more than one tensor component measurement) separated by a base-line in space. By differentiating their outputs it is possible to derive a value proportional to a magnetic gradient along the base-line, provided that the magnetometers' sensitivity axes are aligned with sufficient accuracy. The nature of a single magnetometer is not important in this case and may vary from cesium vapour or fluxgate devices operating at room temperature, to SQUIDs-based fluxmeters operating at liquid helium or liquid nitrogen temperatures.

A technique known functionally as torque or force magnetometry [1], utilises a cantilever sensitive element as a primary sensor. Its mechanical displacements from an equilibrium position are caused by an applied magnetic field and are measured by a secondary transducer (normally of a capacitive or an inductive nature). The method described here is similar to this torque (force) magnetometry, but is better suited for measuring magnetic gradients. The corresponding sensor hardware has been developed as the magnetic counterpart of a novel cryogenic gravity gradiometer aimed at mobile (airborne and shipborne) applications, which has been under development in New Zealand since 1996 [2]. It is designed to be used at low temperature in order to reduce thermal noise and increase mechanical stability. Superconductivity and SQUIDs may be employed to enable the sensor technology to reach its ultimate performance.

The goal of this paper is to briefly introduce the theoretical principle which underlies the method under consideration, and to estimate the potential sensitivity of the corresponding sensor. The last problem is reduced to a well-known one which is to determine the least detectable displacement of a linear harmonic oscillator driven by thermal noise [3].

2. Basic principle and noise characteristics

The basic principle of the magnetic gradient measurements takes advantage of some specific dynamic properties of a stiff metallic current carrying string (wire) fixed at both ends. By "string" no practical limitation as to material or to its cross-section is implied: an elongated object having one long dimension and two much shorter ones, which is capable of carrying an electric current, of being transversely deflected by magnetic forces and of providing a restoring force, can be used as a magnetic gradiometer [4]. Comparatively recently a similar technique has been described in [5] which relates exclusively to the problem of alignment of quadrupole magnets in particle accelerators.

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The string must have a highly uniform mass per unit length distributed along its long dimension. If this condition is satisfied with sufficient accuracy then the string's dynamics can be described by the following force balance equation

$$\eta \frac{\partial^2}{\partial t^2} x(z,t) + h \frac{\partial}{\partial t} x(z,t) - \alpha \frac{\partial^2}{\partial z^2} x(z,t) + \beta \frac{\partial^4}{\partial z^4} x(z,t) = I_s(t)[B_y(0,t) + B_{yz}(0,t)z] + \tilde{f}_L(z,t) \quad (1)$$

with boundary conditions corresponding to the clamped ends of the string, i.e. $x(0,t) = x(l,t) = 0$. Here l is the length of the string and $x(z, t)$ is the displacement of the string in one of the orthogonal directions transverse to z direction which is chosen to point along the string's length.

In Eq. (1), η denotes the string's mass per unit length, h is the friction coefficient per unit length, α and β are some positive constants which determine the restoring force per unit length of the string. The first term in the right side of Eq. (1) represents the Ampere force per unit length acting upon the string in an external nonuniform magnetic field $B(z, t)$ provided that there is a current $I_s(t)$ in the string. The quantity B_{yz} denotes the gradient of the magnetic induction component B_y along z direction. The second term in the right side of Eq. (1) represents the Langevin random force per unit length acting upon the string due to its interaction with the thermostat having the absolute temperature T, with the following correlation function

$$\langle \tilde{f}_L(z_1,t_1) \tilde{f}_L(z_2,t_2) \rangle = 2k_B T h \delta(z_1 - z_2) \delta(t_1 - t_2) \quad (2)$$

where $k_B = 1.4 \times 10^{-23}$ J/K is the Boltzmann constant and $\delta(x_1 - x_2)$ is the delta function.

The fixed boundary conditions imposed upon the string mean that the general solution of Eq. (1) can be represented as an infinite sum over the string's discrete number of eigenfunctions which meet the boundary conditions

$$x(z,t) = \sum_{n=1}^{\infty} c_x(n,t) \sin\left(\frac{\pi n}{l} z\right) \quad (3)$$

where $c_x(n, t)$ is an amplitude of the string's displacement in x-direction for a particular eigenmode n ($n = 1, 2, 3, \ldots$).

By substituting Eq. (3) into Eq. (1) and by multiplying both sides by $\sin(\pi n z/l)$, and then by integrating both sides over z from 0 to l, one can obtain the master-equation for $c_x(n, t)$

$$\frac{d^2}{dt^2} c_x(n,t) + \frac{2}{\tau} \frac{d}{dt} c_x(n,t) + \omega_n^2 c_x(n,t) = \frac{2}{\pi n} [1 - (-1)^n] \frac{1}{\eta} I_s(t) B_y(0,t) - (-1)^n \frac{2l}{\pi n} \frac{1}{\eta} I_s(t) B_{yz}(0,t) + \frac{2}{\eta l} \int_0^l dz \tilde{f}_L(z,t) \sin\left(\frac{\pi n}{l} z\right) \quad (4)$$

Eq. (4) describes a conventional forced harmonic oscillator with the relaxation time τ and a particular resonant frequency ω_n. It can readily be seen that for $n = 2, 4, 6, \ldots$ the vector component of the magnetic field measured disappears from the right side of Eq. (4). This means that tensor characteristics of a magnetic field can be determined by measuring the displacements of a string which correspond to even eigenmodes only.

By using Eqs. (2) and (4) it is straightforward to derive the least detectable magnetic gradient which is limited by the fundamental thermal noise driving force in the right side of Eq. (4). This yields

$$(\text{grad B})_{\min} = \frac{2\pi n}{i_s} \sqrt{\frac{\eta}{l^3} \frac{k_B T}{\tau \tau^*}} \quad (5)$$

where i_s is an amplitude value of the current in the string, t^* is a measurement time and the condition $\tau^* \geq \tau$ is implied. It is also assumed that the measured magnetic gradients are quasi-static ones, and the string is pumped with an alternating current $I_s(t)$ tuned to one of the even eigenmodes of the string ($n = 2, 4, 6, \ldots$). As depicted in Eq. (4), the string is excited at that eigenmode in the presence of a quasi-static magnetic gradient. The corresponding mechanical displacements of the string can be measured by the use of an inductive technique with an instrumental noise envelope of less than $\pm 10^{-12}$ m, per 1 s measurement interval [6].

The following numerical figures have proven to be feasible for practical use: $T = 77$ K, $n = 2$, $i_s = 0.5$ A, $\eta = 3.3 \times 10^{-5}$ kg/m, $l = 0.25$ m, $\tau \sim \tau^* = 1$ s. By substituting the above parameters into Eq. (5) one can obtain the following estimate: $(\text{grad B})_{\min} = 38 \times 10^{-12}$ T/m $=$ 0.038 gamma/m. The above estimate does not depend upon a particular mechanical displacement detection scheme which could vary from using a high-frequency heterodyne-type modulation–demodulation technique in combination with a normal conductive string [6], to well-established SQUIDs-based mechanical displacement sensors. In all cases the use of a superconducting string would improve the obtained estimate by at least an order of magnitude by putting a stronger current into the string and by reducing its mass per unit length.

In the sensor which has been developed to date the string is simply pumped by an additional carrier-frequency current and is inductively coupled to a resonant bridge tuned to the carrier frequency (approx. 3 MHz). This frequency is chosen to be far enough from the string's effective mechanical bandwidth which is normally limited to a few kHz even for very stiff strings. Then mechanical displacements of the string give a low-frequency modulation (at the rate of the second eigenmode of the string which normally lies in between 600 and 1000 Hz) of the carrier signal. A double-lock-in scheme provides firstly amplification and detection of the carrier signal and secondly amplification and detection of the low-frequency envelope with an amplitude proportional to the magnetic gradient measured.

The principle of operation allows the sensor hardware to be highly immune to vector components of an external magnetic field and also to be immune to any kind of mechanical movement. Neither the use of any of magnetic materials nor a passive or active shielding of any part of the sensor hardware are required for SQUIDs-free designs. It can also provide a linear output in the dynamic range of tens of thousands of gamma per metre.

3. Discussion

Instead of using a pair of different magnetic field sensors, a single sensitive element can be used for measuring directly a magnetic gradient (in principle it is possible to measure two magnetic gradients simultaneously, by a single string). The simple geometry of a stretched string allows the sensor to have a well-defined reference frame for benchmarking collected data. The size and weight of the sensor hardware and the flexibility of its intrinsic design make it highly adaptable and cost effective for a range of commercial uses, in particular, for mobile surveying. Portable off-the-shelf cryogenic facilities are readily available for maintaining the sensor at low temperature and would not increase its size and weight to the level which may prevent use of the technology for mobile deployment. Multiple laboratory testing has proven that the accuracy of measurements, which can be obtain with this new type of instrument, is sufficient enough for high quality direct measurements of magnetic gradients.

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Biography

Alexey Veryaskin was born in Tashkent, Uzbekistan (formerly USSR) in 29 October, 1951. He obtained his MSc degree in Electronics and Computer Technique from Penza Polytechnic Institute (city of Penza, Russia) in 1973. PhD in Physics and Mathematics was obtained by him from the Scientific Research Centre for Physical, Technical and Radiotechnical Measurements (Moscow, Russia) in 1982. From 1975 to 1988, he was employed at the Sternberg State Astronomical Institute, Moscow University, where he was working on SQUIDs based sensor technologies applied to precise gravitational measurements. From 1992 to 1995 he was employed at Physics and Applied Physics Department of the University of Strathclyde (Glasgow, UK) where he was working under the aegis of European Space Agency on some aspects of the STEP space project. After moving to New Zealand in 1995, he became Principal Scientist and Project Leader at Gravitec Instruments Ltd. — a UK based research and development company specialising in cryogenic advanced sensor technologies. His current fields of interest are gravity and magnetic gradiometry for natural resource exploration, space and terrestrial navigation, natural hazards prevention.